Ultrasoft Fermion Mode at Finite Temperature and Density

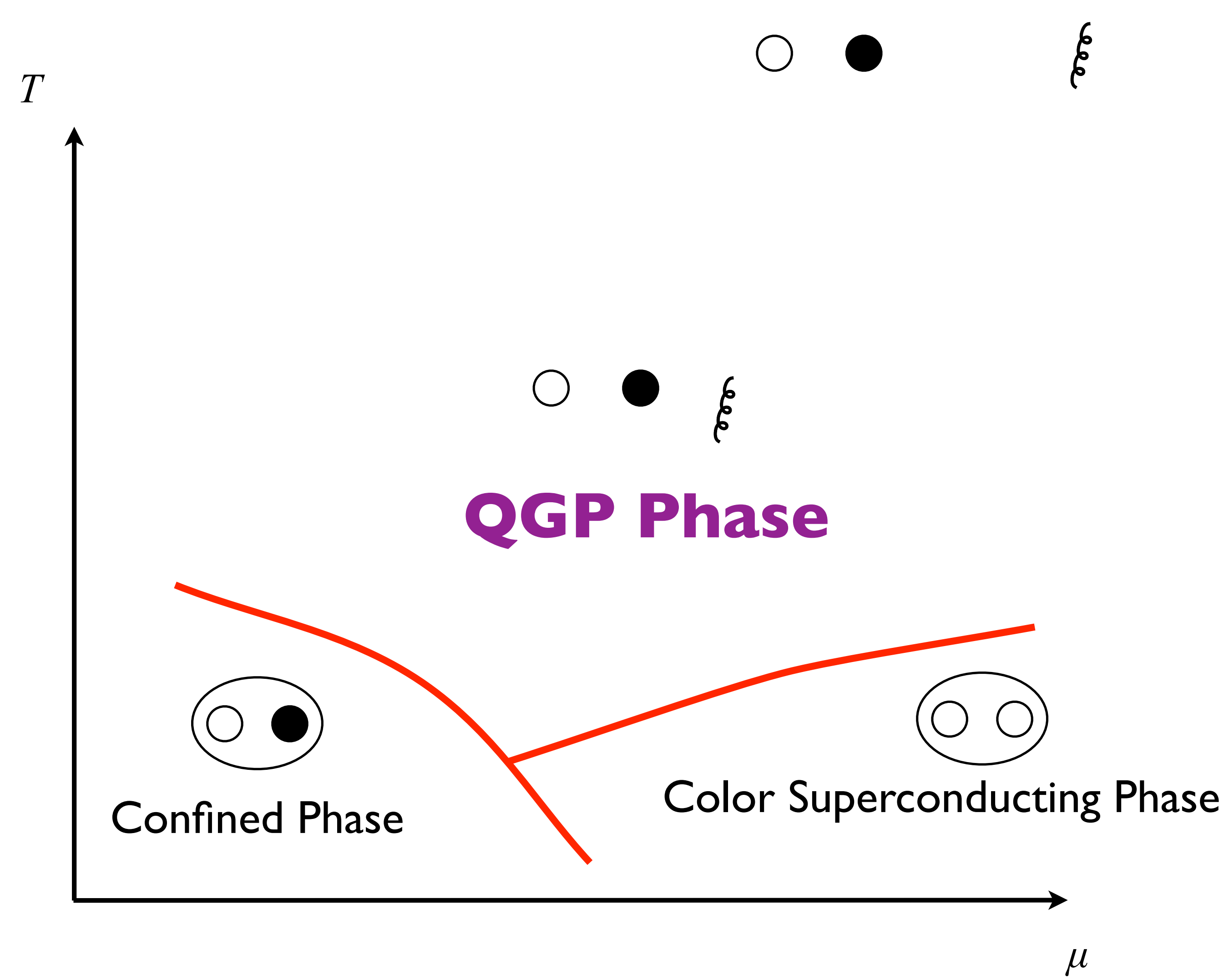
D. Satow (Kyoto Univ. → RIKEN)

Collaborator: J. P. Blaizot (Saclay-CEA)

Introduction

Quark-Gluon Plasma (QGP)

Basic degree of freedom: quark, gluon



Introduction

Motivation: Clarifying **quark spectrum** in QGP phase.

In general, particle picture in medium is complicated.

- How many excitation modes?

(Emergence of collective excitation. cf: plasmon, phonon in condensed matter system)

- How are the dispersion relation, damping rate, strength?

(Since there is no Lorentz symmetry, the Lorentz-noncovariant dispersion relation is allowed.)

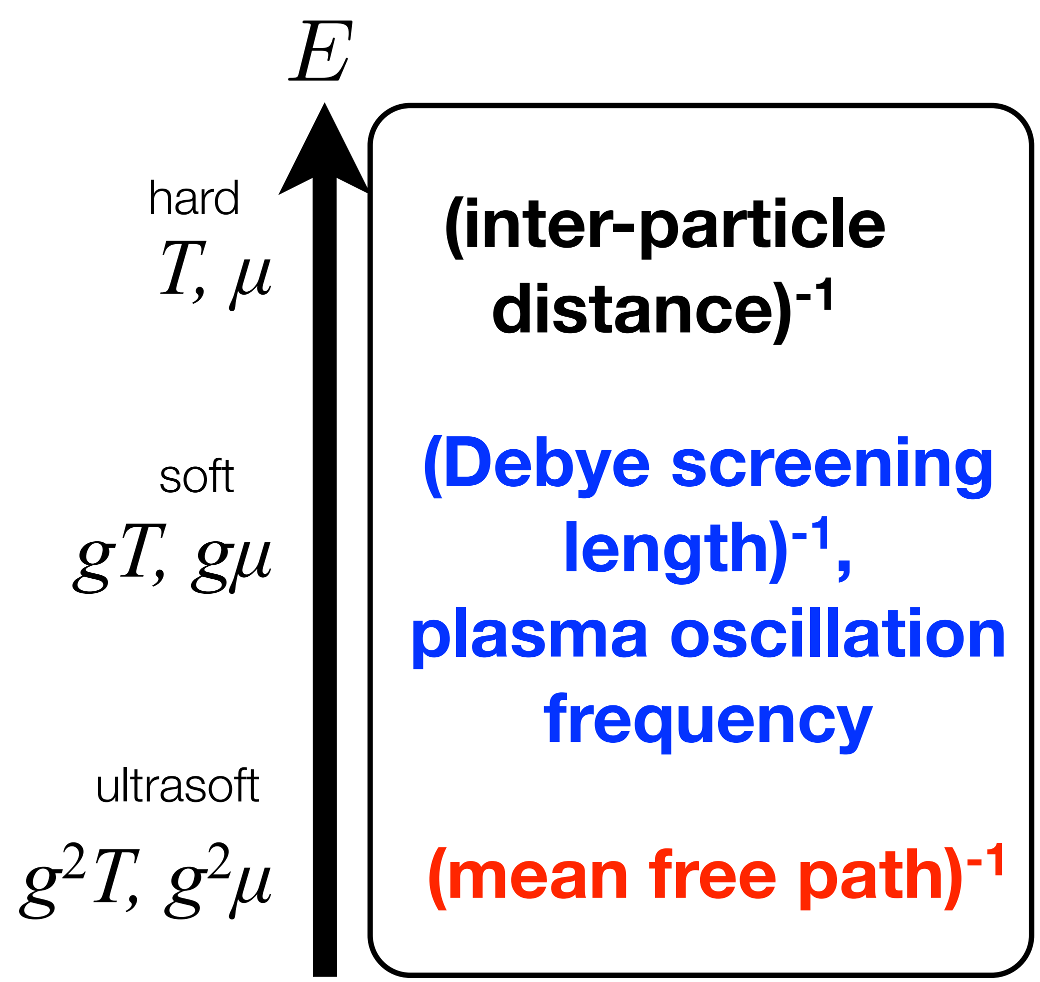
$$\cancel{(\omega^2=p^2+m^2)}$$

Energy hierarchy

This is nontrivial task;
Free-particle picture is generally **invalid** even at weak coupling ($g << 1$).

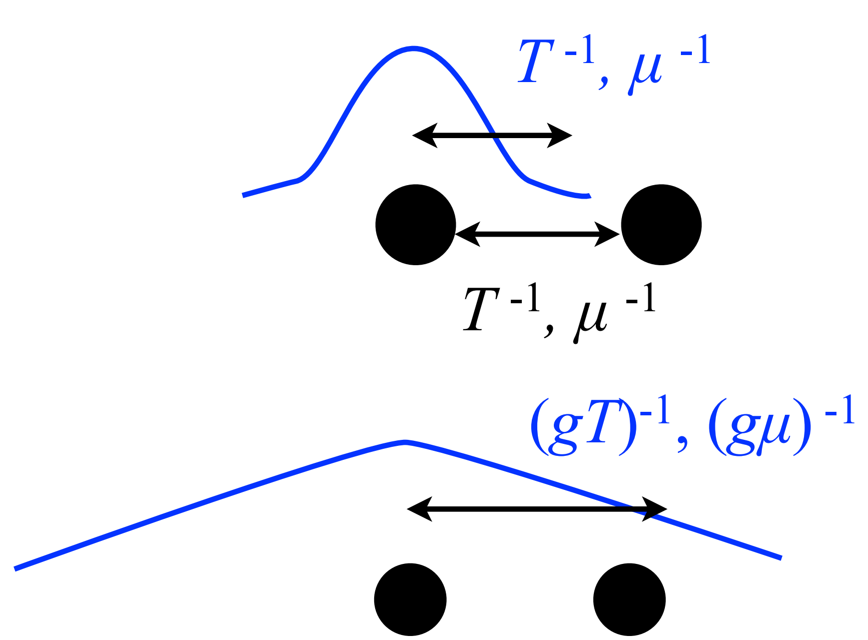
Yukawa model, QED/QCD

energy hierarchy at $T, \mu >> \Lambda_{QCD}, m$



Because

- many-body effect becomes non-negligible when (energy) ~ $gT, g\mu$

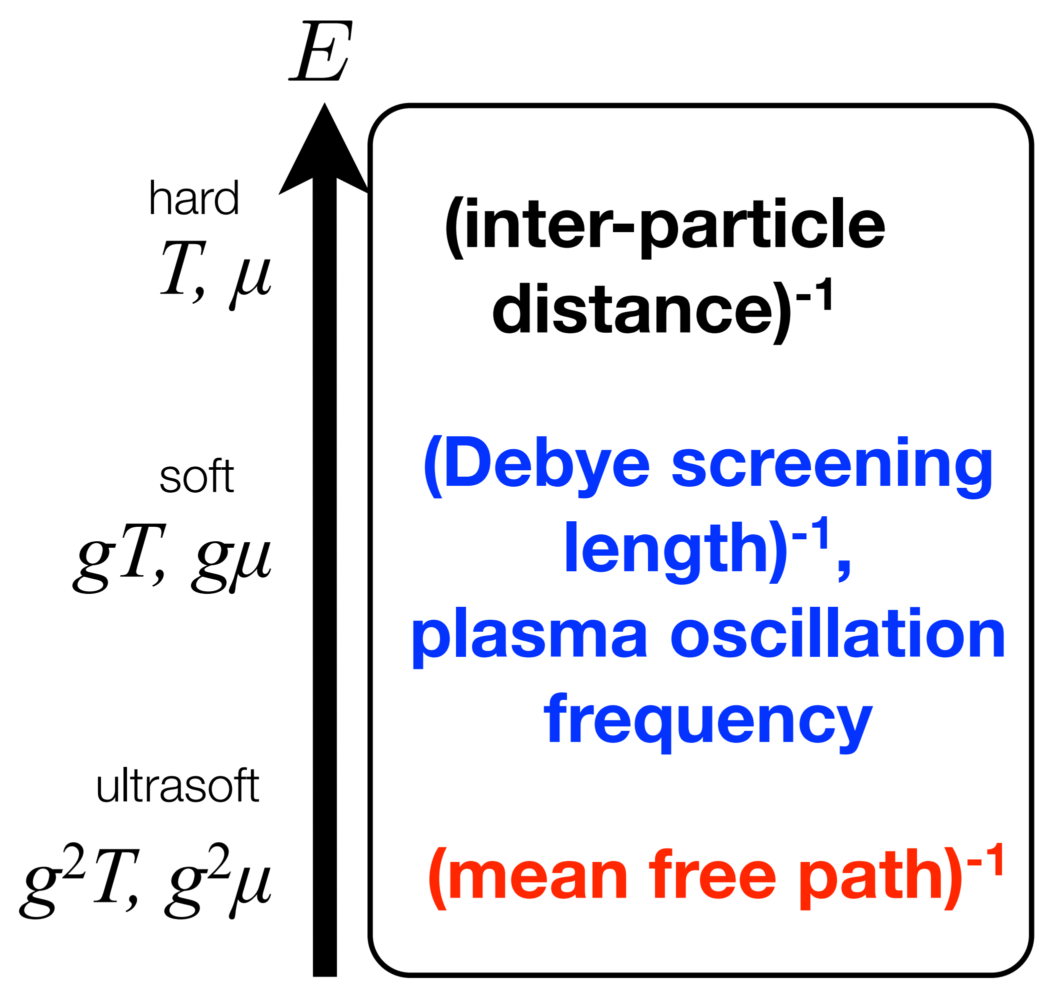


Energy hierarchy

This is nontrivial task;
Free-particle picture is generally **invalid** even at weak coupling ($g <<1$).

Yukawa model, QED/QCD

energy hierarchy at $T, \mu >> \Lambda_{QCD}, m$



Because

- many-body effect becomes non-negligible when (energy) ~ $gT, g\mu$
- Furthermore, interaction effect such as collision becomes non-negligible when (energy) ~ $g^2T, g^2\mu$

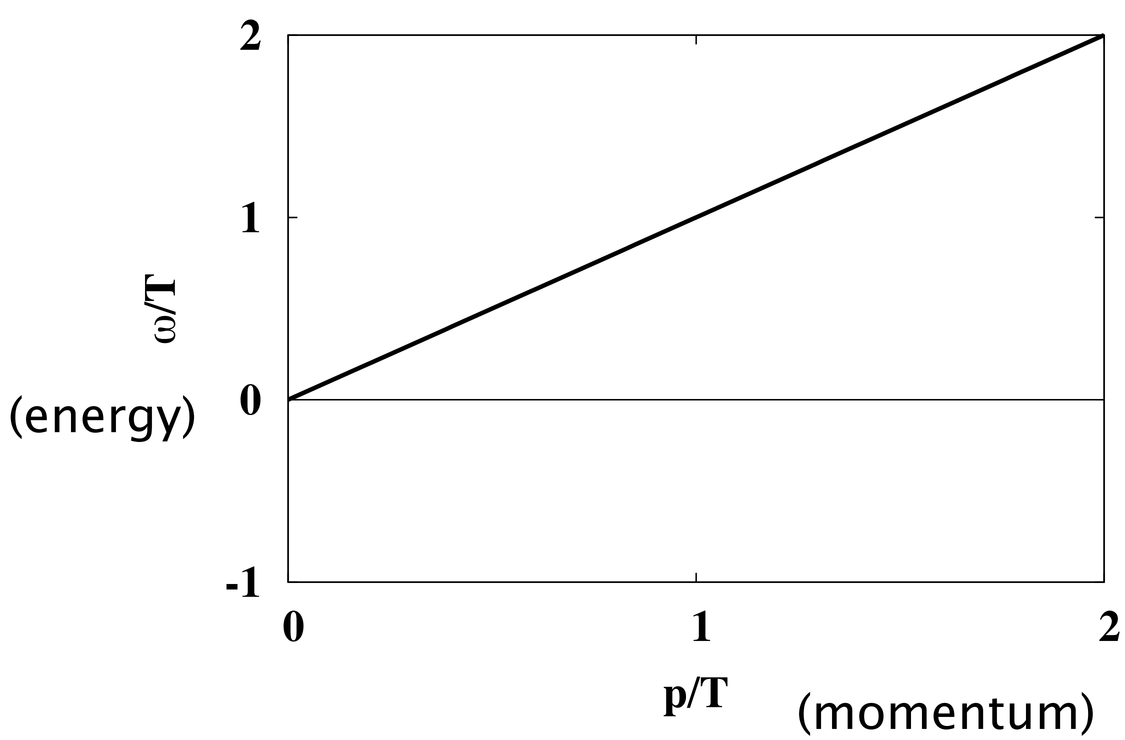
(<u>mesoscopic scale</u>)

Ultrasoft scale is **not well investigated** even at weak coupling ($g <<1$).

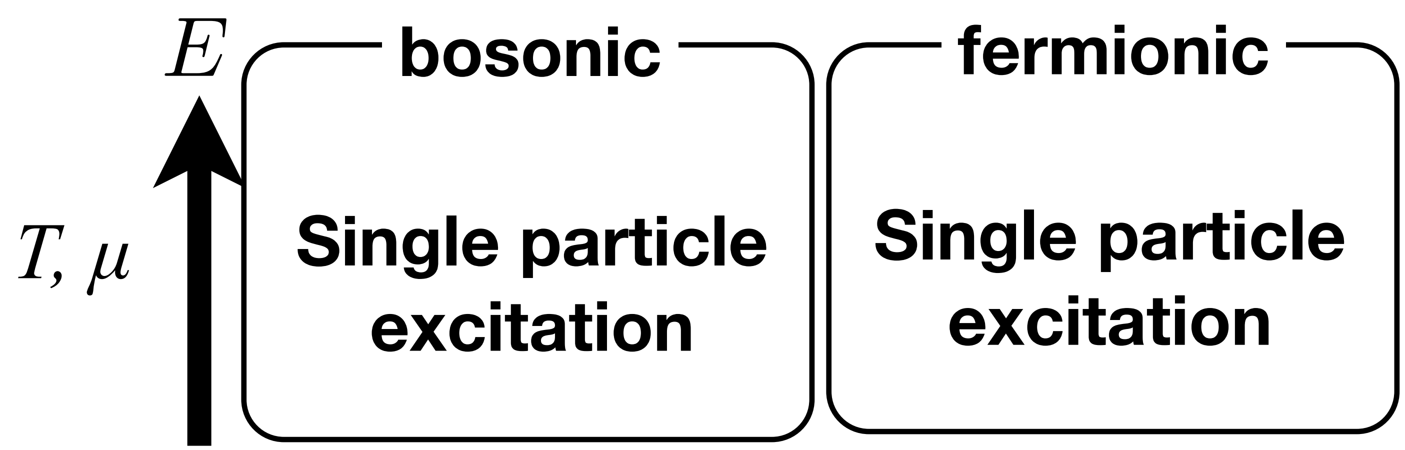
Hard scale ($p \sim T, \mu$)

Therefore the interaction effect is suppressed by g, and thus the free-particle picture is valid.

dispersion relation in the free limit： $\omega=|p|$



dispersion relation in fermionic sector



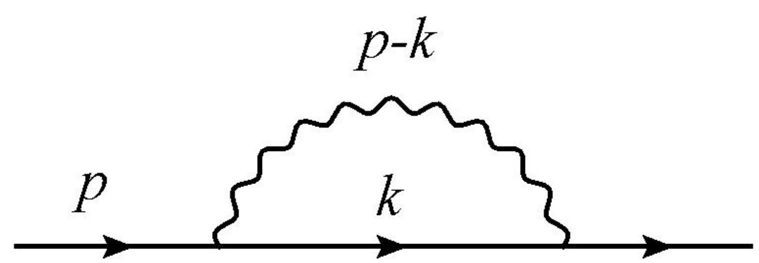
Soft scale ($p\sim gT,\ g\mu$)

E. Braaten and R. D. Pisarski, Nucl. Phys. B **337**, 569 (1990)

Hard Thermal Loop (HTL) approximation

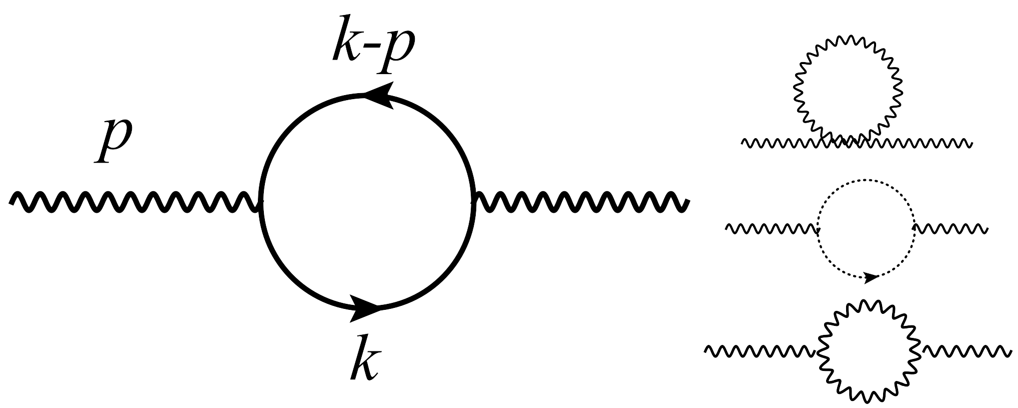
- valid when $p\sim gT,\ g\mu$.

Fermion self-energy

$\Sigma(p)=$ 

- 1-loop diagram

Boson self-energy

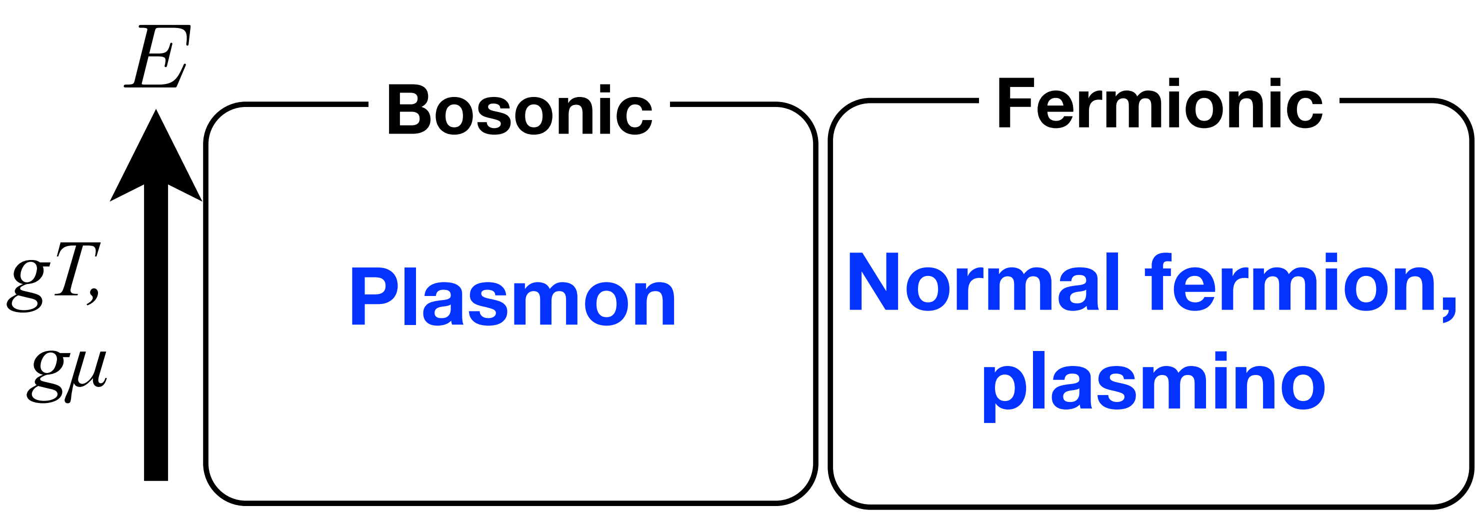
$\Pi^{\mu\nu}(p)=$ 

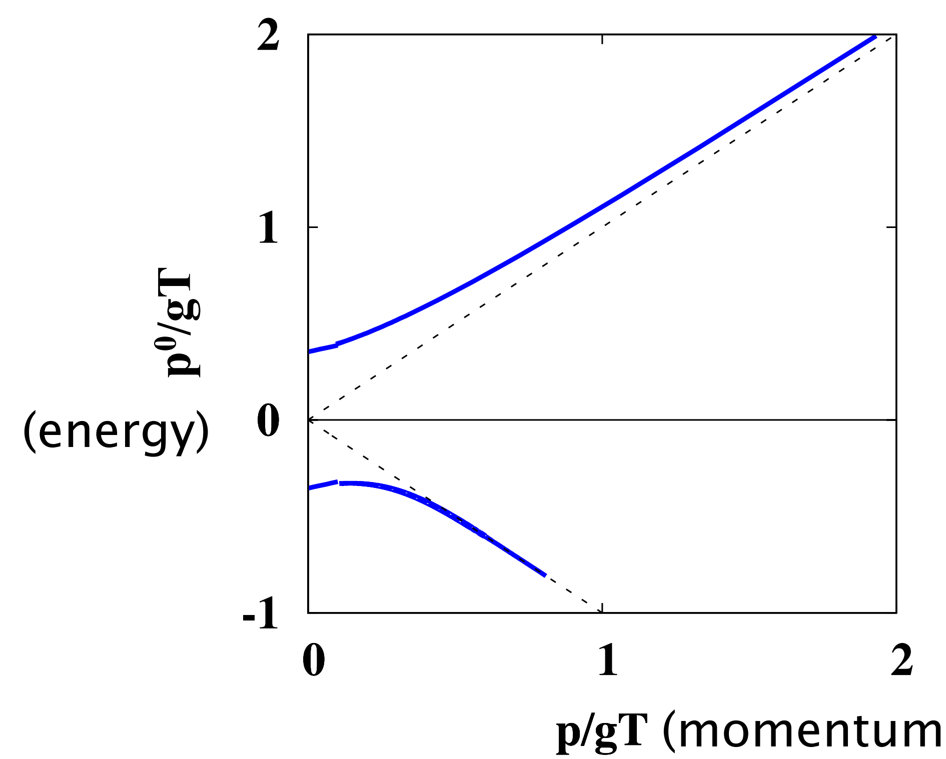
Result

H. A. Weldon, PRD **26**, 1394 (1982), 2789 (1982)

Large medium effect

Collective modes appear.

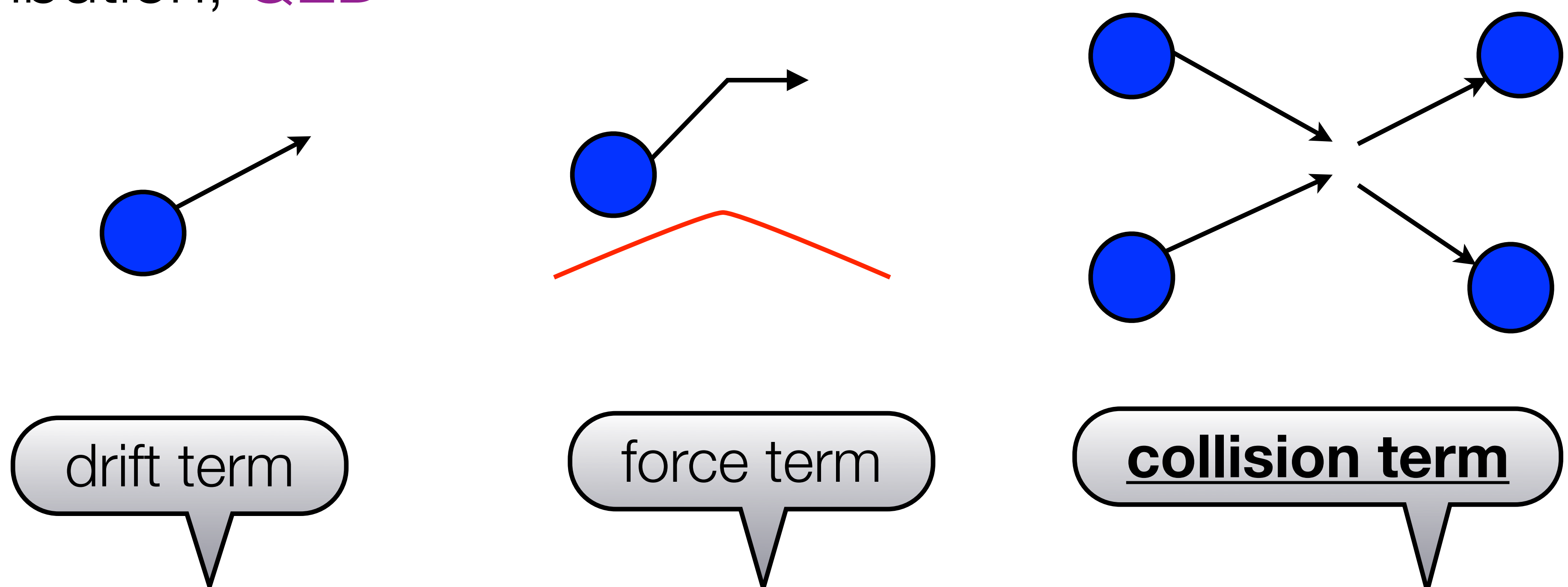




dispersion relation in fermionic sector

Vlasov equation

e.g. : fermion distribution, QED



Boltzmann eq.: $(v \cdot \partial_X + g(\boldsymbol{E}(X)+\boldsymbol{v}\times\boldsymbol{B}(X)) \cdot \partial_{\boldsymbol{k}})n(X, \boldsymbol{k})=C[n]$

$n(X, \boldsymbol{k})$: distribution function of fermion
$v = (1, \boldsymbol{k}/|\boldsymbol{k}|)$: 4-velocity, $\boldsymbol{E}(X)$, $\boldsymbol{B}(X)$: field strength

Vlasov equation

The collision term in Boltzmann eq. is neglected.

Vlasov Eq.: $(\boldsymbol{v}\cdot\partial_X+g(\boldsymbol{E}(X)+\boldsymbol{v}\times\boldsymbol{B}(X))\cdot\partial_{\boldsymbol{k}})n(X,\boldsymbol{k})=\cancel{C[n]}$

$$\partial_X=O(gT)>>C[n]=O(g^2T)$$

Self-energies calculated from Vlasov eq. coincides with that from HTL approximation.

J. P. Blaizot and E. Iancu, PRL **70**, 3376 (1993)

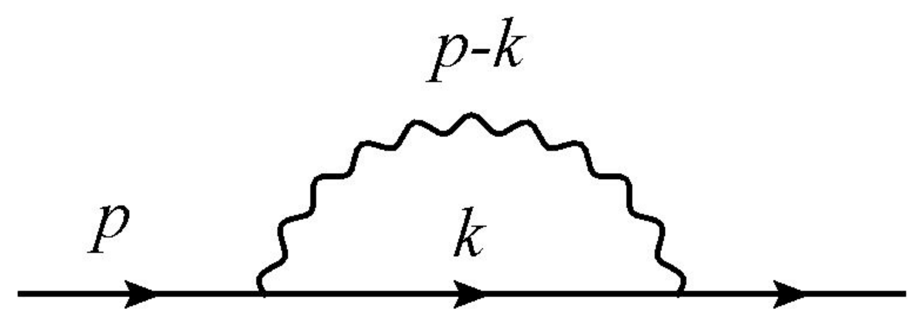
By contrast, the collision term is non-negligible at <u>ultrasoft</u> region ($\partial_X=O(g^2T, g^2\mu)$)!

Ultrasoft scale ($p \sim g^2 T,\ g^2\mu$)

Reflecting this fact, the HTL approximation is not applicable in ultrasoft region($p \lesssim g^2 T,\ g^2\mu$).

$p \to 0$ limit can not be taken (pinch singularity)

$$g^2\int\frac{d^4k}{(2\pi)^4}2\pi\theta(k^0)\delta(k^2)(N(k^0)+n(k^0))\frac{\not{k}}{2p\cdot k-p^2}\xrightarrow{p\to 0}\infty$$



Pinch singularity in the computation of the transport coefficient:
S. Jeon, PRD **52**, 3591 (1995)

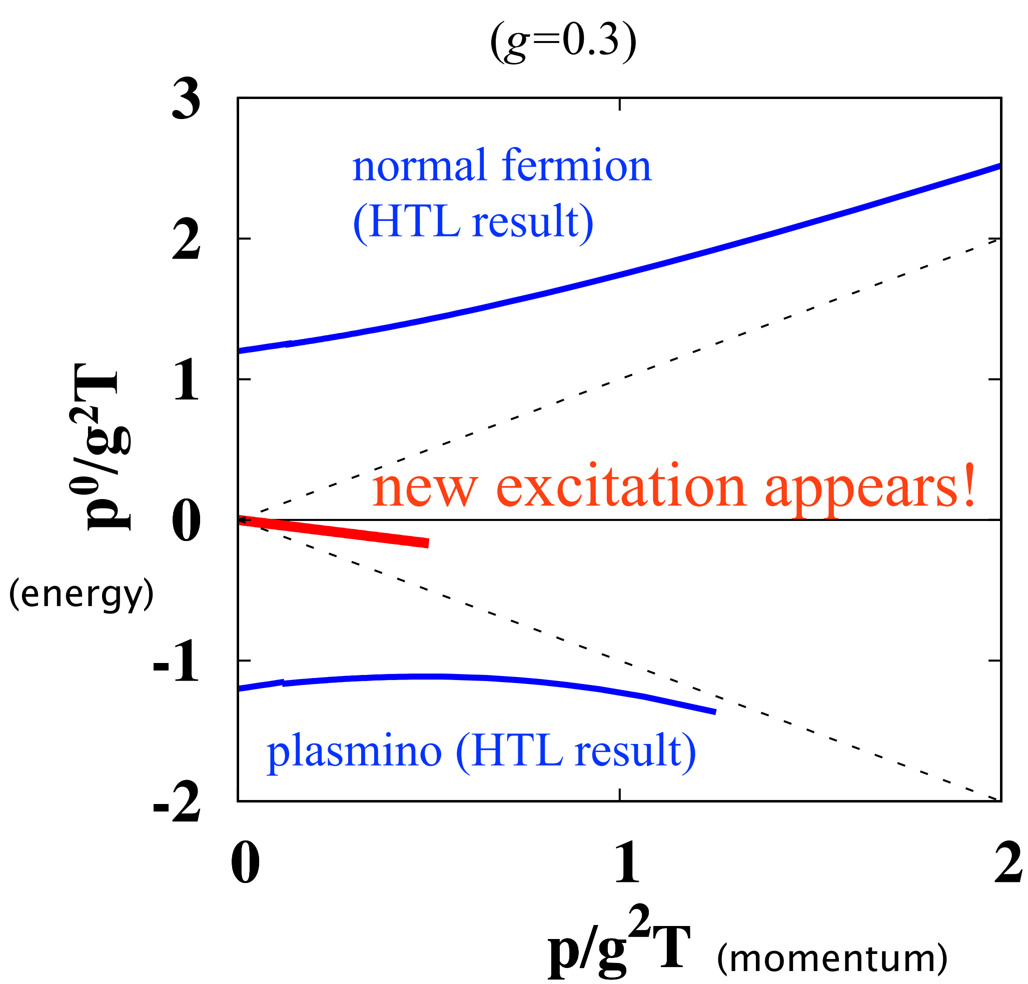
reorganizing perturbative expansion is necessary.

V. V. Lebedev and A. V. Smilga, Annals Phys. **202,** 229 (1990)

Ultrasoft Fermion Mode at μ=0

Resummed perturbation showed the existence of a novel excitation in ultrasoft ($p<<g^2T$) region.

Y. Hidaka, **D. S.**, and T. Kunihiro, NPA **876**, 93 (2012)
D. S., arXiv: 1303.2684 [hep-ph]



dispersion relation in fermionic sector

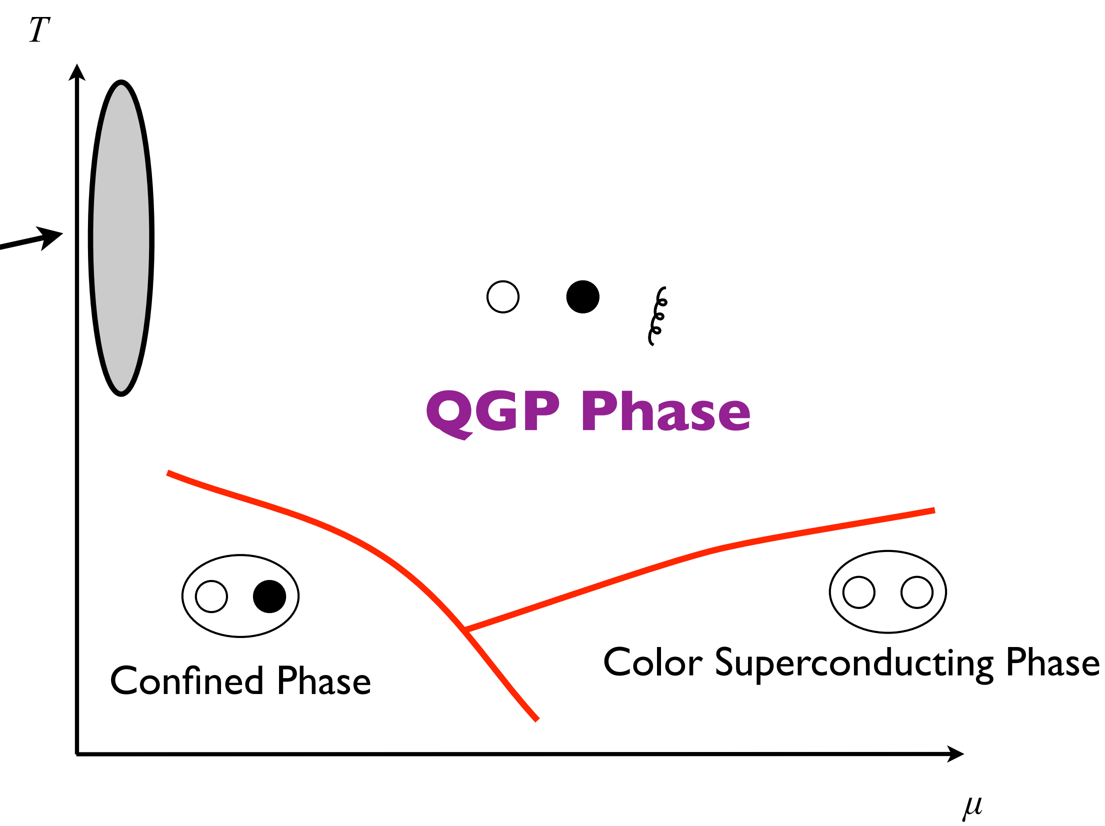
Suggestions:
Resummed perturbation: V. V. Lebedev and A. V. Smilga, Annals Phys. **202,** 229 (1990).
Schwinger-Dyson eq.: M. Harada and Y. Nemoto, PRD **78**, 014004 (2008), S. X. Qin, L. Chang, Y. X. Liu, and C. D. Roberts, PRD **84**, 014017 (2011).
NJL model: M. Kitazawa, T. Kunihiro and Y. Nemoto, PLB **633**, 269 (2006).

Excitations at μ=0

E	**Bosonic**	**Fermionic**
hard T	**Single particle excitation**	**Single particle excitation**
soft gT	**Plasmon**	**Normal fermion, plasmino**
ultrasoft g^2T	**Hydro mode**	**Ultrasoft fermion mode**

Ultrasoft Fermion Mode at Finite μ

We analyzed this region.
(High-T, μ=0)



How about in other region？

Resummed Perturbation (<u>T=0, finite-μ</u>)

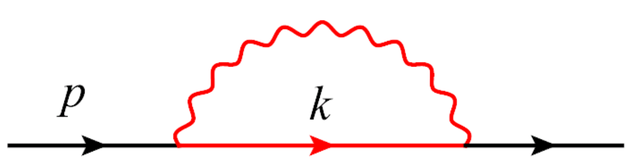
resum the masses due to density effect (m_f, m_b=$O(g\mu)$) and the damping rates (ζ_f, ζ_b=$O(g^4\mu)$)

(Yukawa model, for simplicity.)

m_f, ζ_f

m_b, ζ_b

→Pinch singularity is regularized.



$$g^2 \frac{\not{k}}{2p\cdot k + \delta m^2 + 2i\zeta k^0} \xrightarrow{p\to 0} O(g^0)$$

$\delta m^2 = m^2{}_b - m^2{}_f$, $\zeta = \zeta_f + \zeta_b$

Result

Dispersion relation	$\mathrm{Re}\,\omega = -\frac{p}{3} + \frac{g^2\mu}{4\pi^2}$

This term breaks the expansion condition ($p << g^2\mu$), so the result is not reliable.
<u>The mode does not exist</u> in the corresponding region to the high-T case.

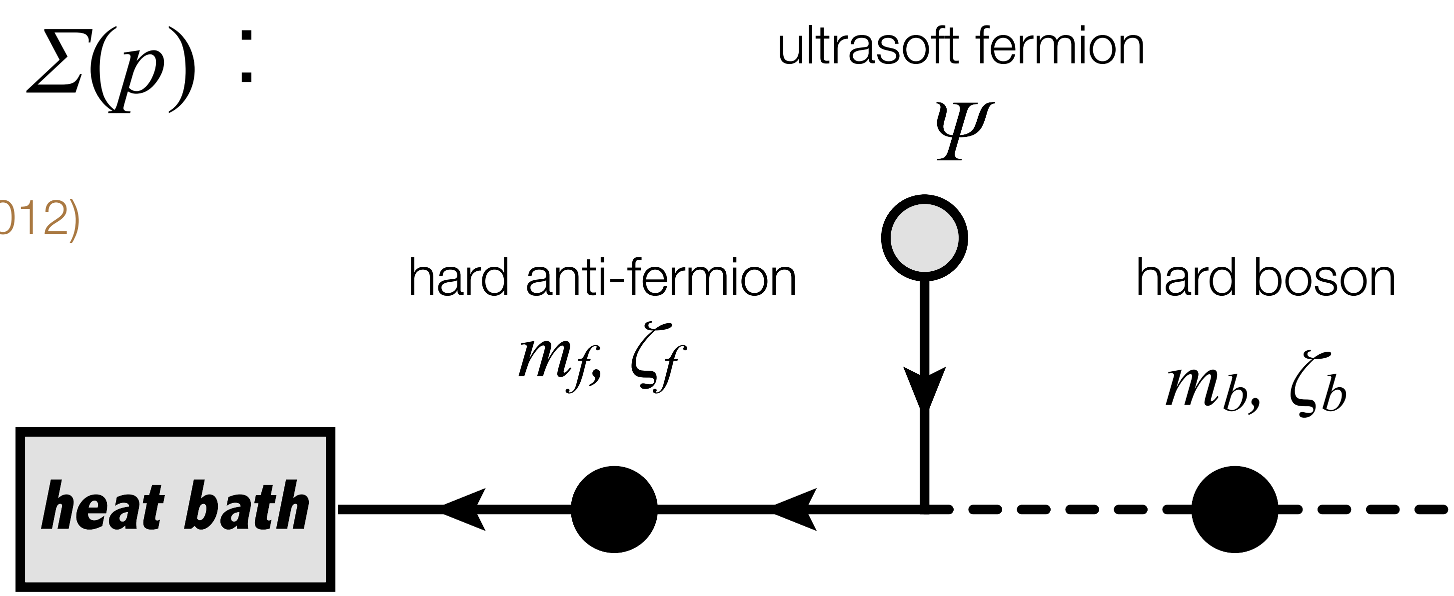
Argument based on dynamics

To satisfy the pole condition ($\not{p} - \Sigma(p) = 0$), $\Sigma(p)$ should be small when $p=0$.

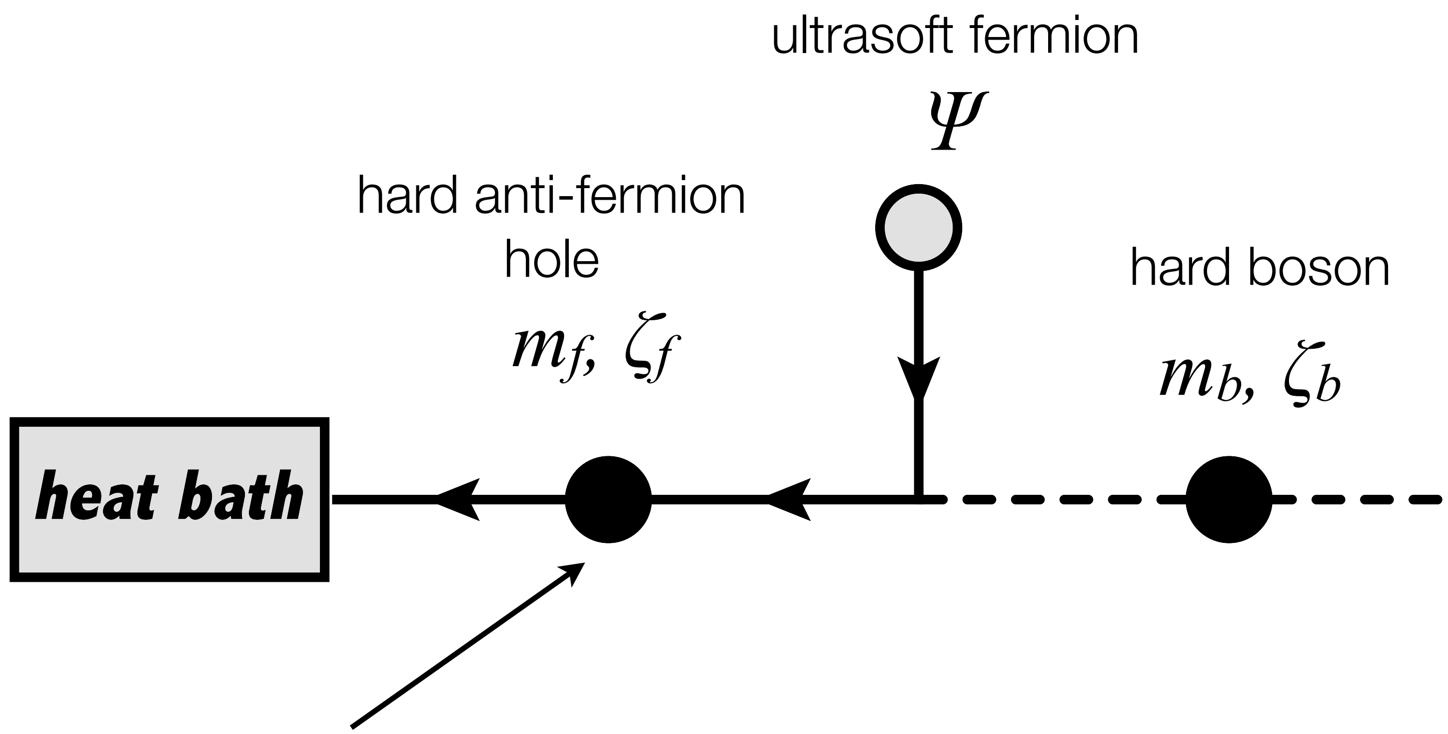
Process contributing to $\Sigma(p)$:

D. S. and Y. Hidaka, PRD **85**, 116009 (2012)

D. S., arXiv: 1303.2684 [hep-ph]



Argument based on dynamics



The contribution of anti-fermion cancels that of hole in μ=0 case.

(Charge Symmetry)

At finite μ, there are no anti-fermion in the heat bath, and $\Sigma(p)$ becomes large.

The pole does not exist.

Argument based on symmetry

Chiral, Charge symmetry

H. A. Weldon, PRD **61**, 036003 (2000)

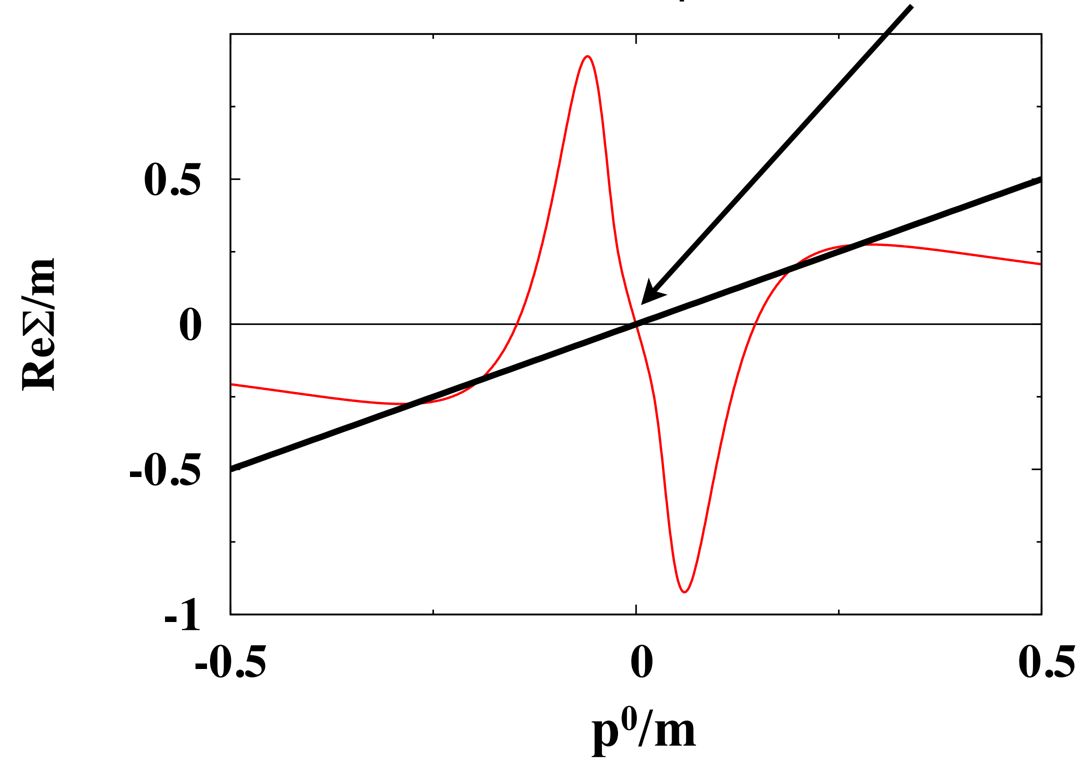
$$S^R(p^0, \mathbf{0}) = -\frac{\gamma^0}{p^0 - \Sigma(p^0, \mathbf{0})}$$

(ReΣ is odd.)

ReΣ is odd, so p^0-ReΣ is zero at p^0=0.

(ImΣ is small enough.)

pole at the origin



Argument based on symmetry

finite μ ⇨ ~~Charge symmetry~~

finite fermion mass ⇨ ~~Chiral symmetry~~

M. Kitazawa, T. Kunihiro, K. Mitsutani and Y. Nemoto, PRD **77**, 045034 (2008).

~~**Ultrasoft fermion mode**~~

Persistency of ultrasoft mode

Ultrasoft mode does not exist at finite-μ, T=0.

How large μ kills the mode?

$\mu \sim T\,?,\ gT\,?,\ g^2T\,?$

naive guess

Resumed Perturbation ($T >> \mu$)

Since $T >> \mu$, the scheme is the same as that in the case of finite-T, $\mu=0$.

V. V. Lebedev and A. V. Smilga, Annals Phys. **202,** 229 (1990)

Resum thermal masses (m_f, $m_b = O(gT)$)

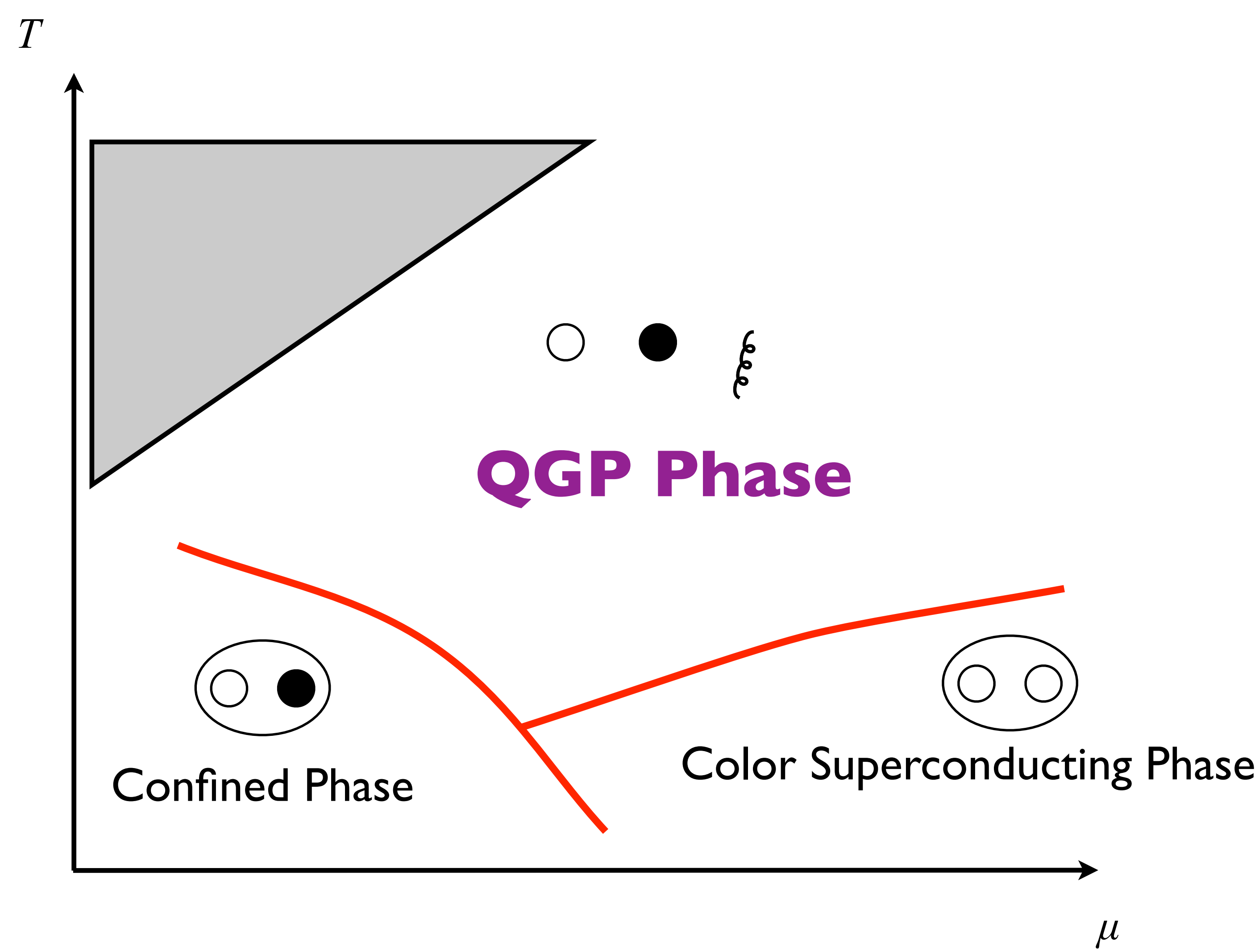
and decay widths (ζ_f, $\zeta_b = O(g^4 T)$).

m_f, ζ_f

m_b, ζ_b

Result

The mode exists as long as ***<u>T >> μ</u>***.



Result

This term does not break the expansion condition ($p<<g^2T$) when $T>>\mu$.

Dispersion relation	$\mathrm{Re}\,\omega = -\frac{p}{3} + \frac{g^2\mu}{36\pi^2}$
Decay width	$\mathrm{Im}\,\omega=\zeta=O(g^4T)$
Residue	$\frac{g^2}{72\pi^2}$

Summary

- We showed that **ultrasoft fermion mode does not exist when μ is large.**

- We showed that **the mode exists as long as $T >> \mu$**.

- We obtained the expressions of the **dispersion relation, decay width, and the residue** for $T >> \mu$.

- We discussed the origin of the mode from the point of view of the **chiral symmetry** and **charge symmetry**.

Back Up

V. V. Lebedev and A. V. Smilga, Annals Phys. **202,** 229 (1990)

Resummed Perturbation (finite-T, μ=0)

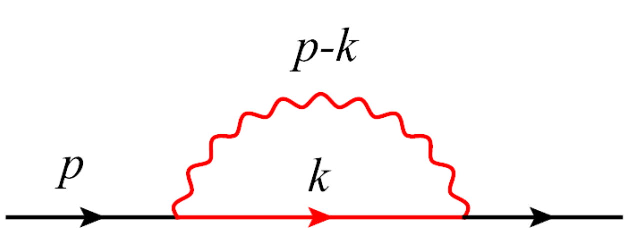
(1) Resum the thermal masses (m_f, m_b=$O(gT)$) and decay widths (ζ_f, ζ_b=$O(g^2T)$ or $O(g^4T)$).

QED (electron)、QCD　　Yukawa model、QED (photon)

m_f, ζ_f

m_b, ζ_b

→Pinch singularity is regularized.



$$g^2 \frac{\not k}{2p\cdot k + \delta m^2 + 2i\zeta k^0} \xrightarrow{p\to 0} O(g^0)$$

δm^2=$m^2{}_b$-$m^2{}_f$, ζ= ζ_f+ζ_b

V. V. Lebedev and A. V. Smilga, Annals Phys. **202,** 229 (1990)

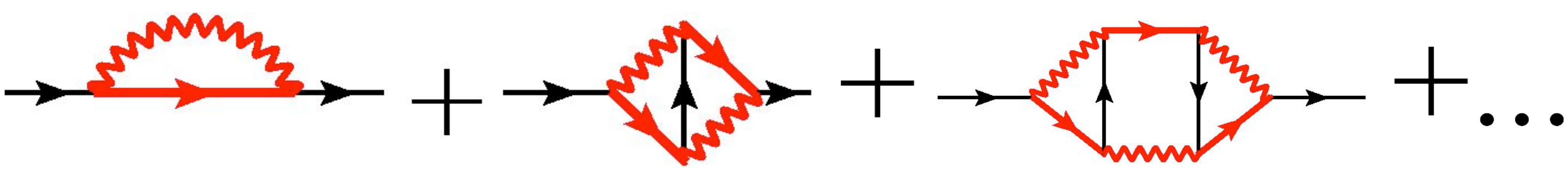
Resummed Perturbation (finite-T, μ=0)

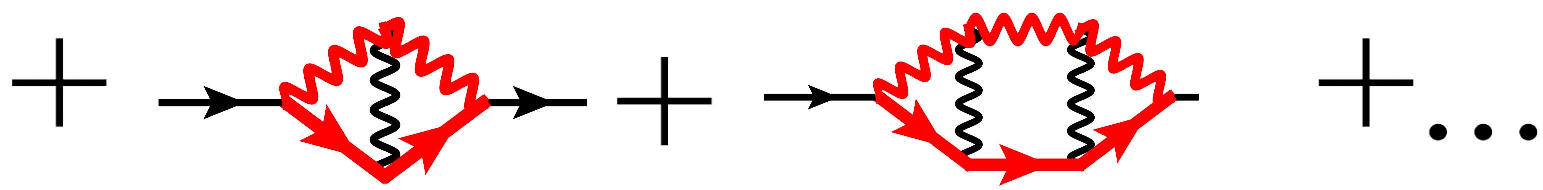
(In the case of QED/QCD)

The singularity is regularized, but **all the ladder diagrams contribute at the same order.**

(#vertex)/(#red line)=1

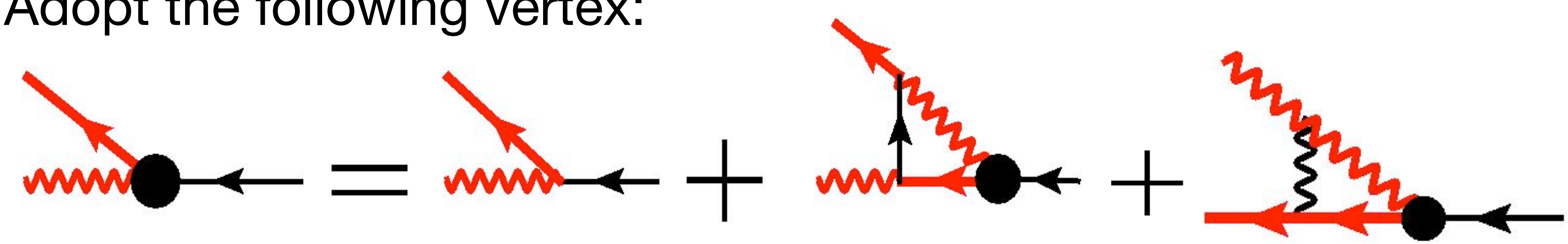
$g^2 \times 1/g^2 = g^0$ $\quad$ $g^4 \times 1/g^4 = g^0$ $\quad$ $g^6 \times 1/g^6 = g^0$





(2) Sum up the Ladder diagrams.

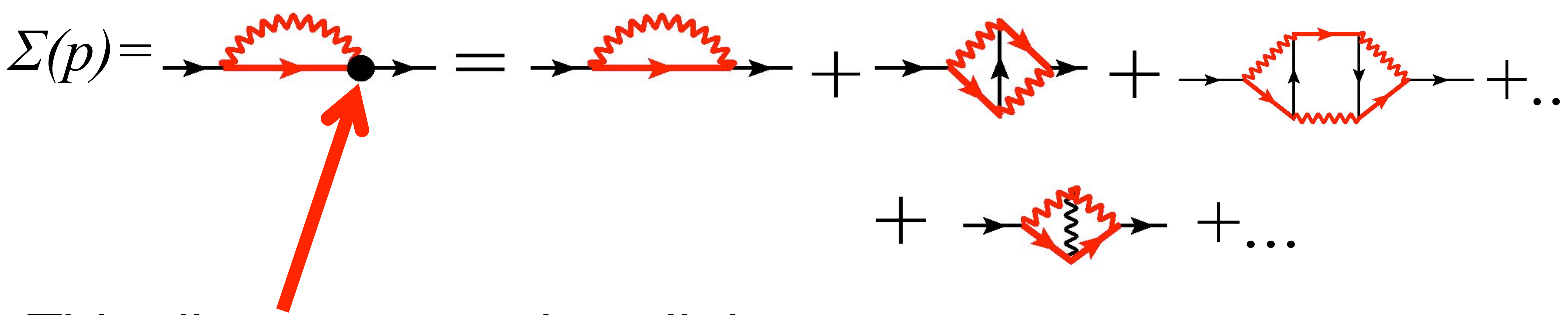
Adopt the following vertex:



V. V. Lebedev and A. V. Smilga, Annals Phys. **202,** 229 (1990)

Resummed Perturbation (finite-T, μ=0)

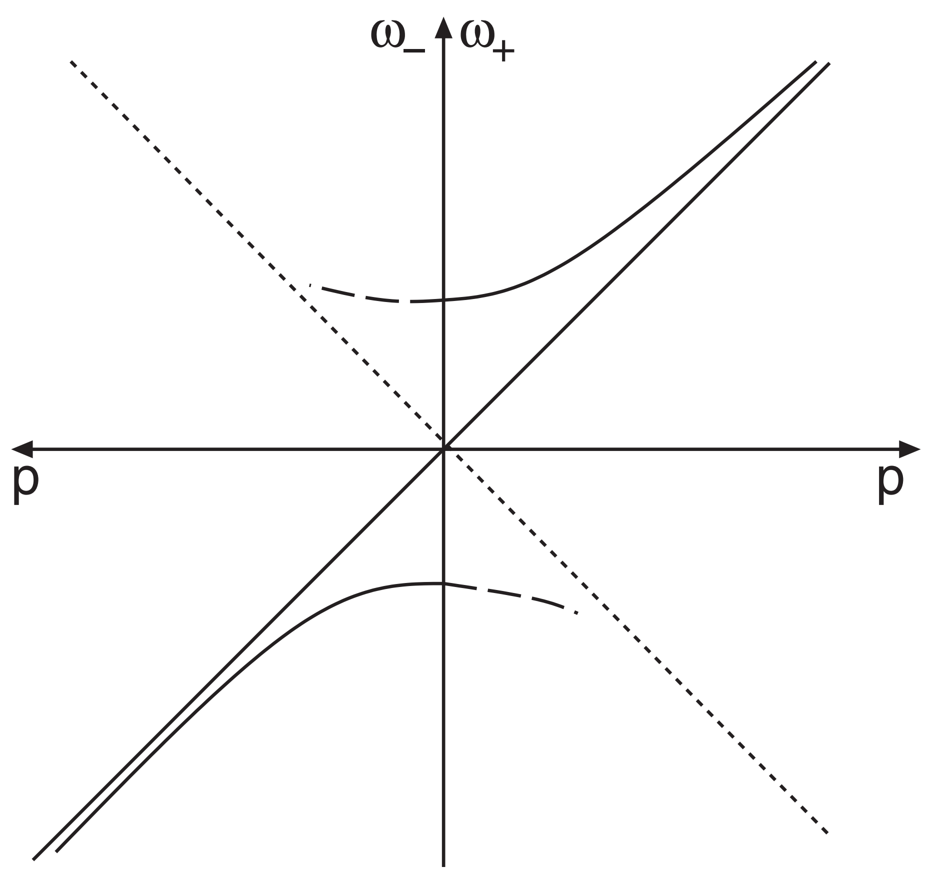
(1), (2)



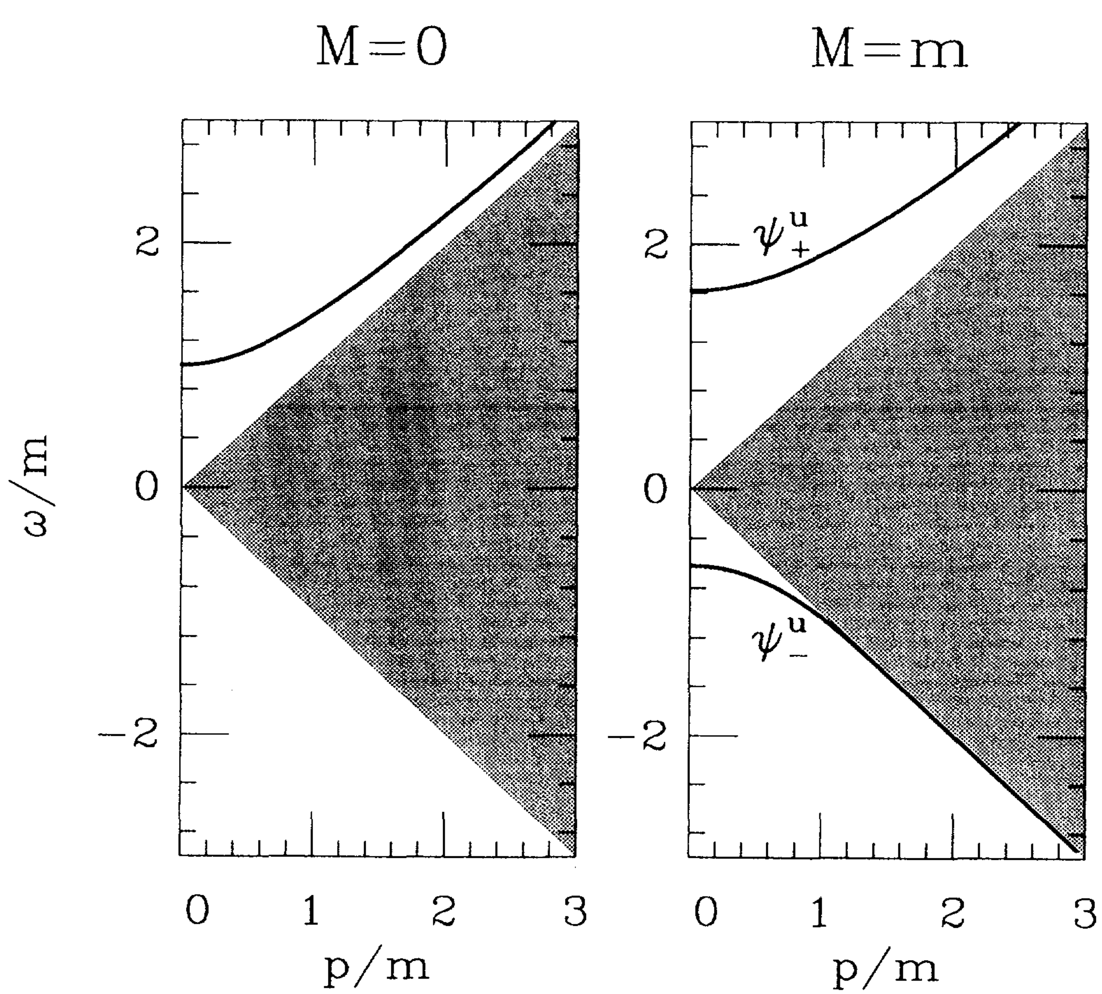
This diagram contains all the contribution at leading order $(O(p/g^2))$.

Interpretation of Plasmino

level repulsion between anti-fermion and hole with boson



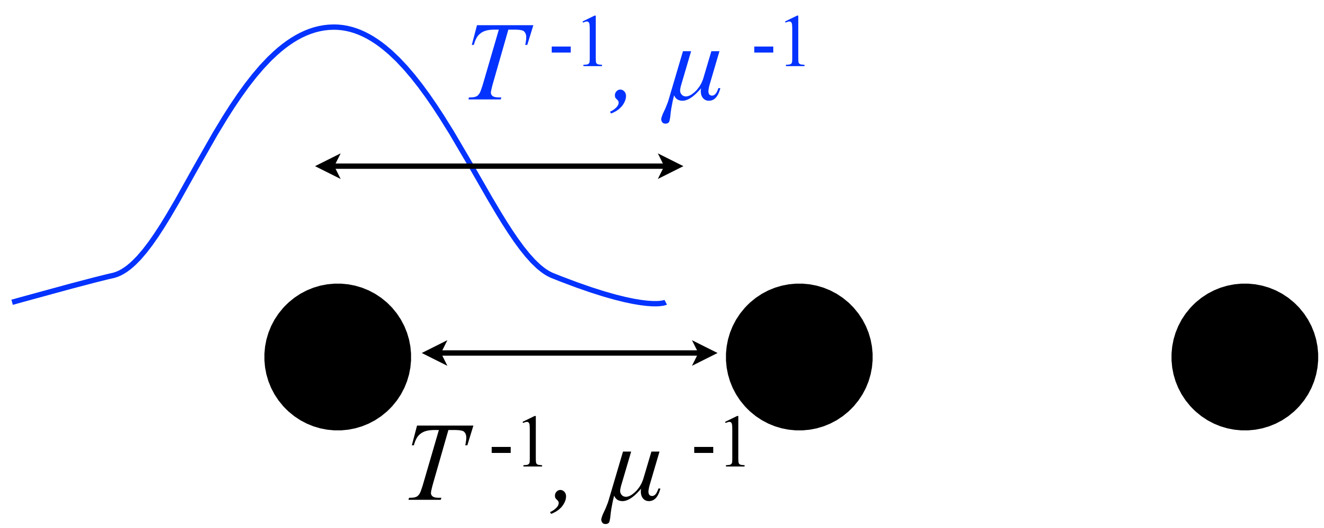
level repulsion between hole and hole with boson



No mixing between hole and anti-fermion!!

Hard scale ($p \sim T, \mu$)

In **hard** scale, the medium effect is small since the scale is the same as the inter-particle distance.



Soft scale ($p \sim gT,\ g\mu$)

In the **soft** scale, we consider the large distance compared with the inter-particle distance, so the medium effect is not negligible.